

PREDICTING STOCKS AVERAGE PRICE: A NEURAL FUZZY APPROACH

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ABSTRACT

This paper is related to the application of adaptive fuzzy system for human expert emulation. The variables taking into account for financial investment (more precisely, stocks transactions, average price and closing price) are used as input-output information for black-box modeling.

The advantage of this method is that thanks to the adaptive fuzzy system it is possible to incorporate to the model the knowledge of an investment advisor. To do it, we have to define the investment advisor's membership functions in the universe of discourse.

Although only three variables are used here, the method proposed is flexible. More variables can be used to obtain a more precise result. Additionally, different membership functions can be taken into account in the modeling process.

Keywords: financial investment, stocks transactions, average price, closing price, artificial intelligence, financial prediction, fuzzy logic.

1. INTRODUCTION

The original motivation and objective of the use of the fuzzy system approach (Sugeno and Tanaka, 1991) was mainly to characterize human behavior in a quantitative way which is based on qualitative knowledge, typically expressed as a set of linguistic IF-THEN statement. Recently, considerable effort has been made to develop the fuzzy modeling approaches in which the available information used to construct fuzzy model is measured by numerical data. Thereby, it provides an alternative and comparable modeling tool to the traditional mathematical methods (Klirk and Yuan, 1995), (Jang, Sun and Mizutani, 1997).

In contrast to the rule-based expert system, a fuzzy model is initially governed by structured knowledge with fuzzy linguistic rules. Then, it can be encoded into numerical framework by fuzzy inference (or identification) in terms of system nonfuzzy (crisp) input-output data. Consequently, the fuzzy model can be effectively applied to continuous function-oriented modeling, estimation, control and optimization. Generally, the fuzzy modeling may be carried out in two ways, the linguistic approach and the fuzzy relation equations under a numerical framework. In this study, only the latter will be investigated.

The evolution of quotation on the stock market is essentially nonlinear time-varying system. A model to predict the behavior of a complex system such as the quotation on the stock market should have the following properties:

- 1.- High nonlinearity.
- 2.- Simple structure.

The former is required to represent the behavior of a complex system which has high nonlinearity. The latter is required to realize easy analysis, design, monitoring, and maintenance of a complex system. In particular, simple structure is connected to efficient structure identification (Tanaka, Sano and Watanabe, 1995). Efficient structure identification is important because a complex system has many predictive variables. In short, a model, which has both properties, is required to identify a complex system. The above requirement is difficult to meet because of the conflicts between the first property and the second one. Therefore, we need to find a model that satisfies both properties as much as possible.

In this work we seek to apply the fuzzy logic to the environment of the Stock Exchange and to predict the average price of the securities that trade on the Stock Exchange. Concretely, we have centered in the companies of Internet, able to provide as much earnings as spectacular losses. For it, we have structured our work in the following parts: in the second section we expose the systems habitually used to value stocks; in the third, we show the problems that think about to value the stocks of the companies of Internet using the traditional systems of valuation, as well as, the intents that have been carried out to apply the neural networks and the fuzzy logic to the valuation of stocks; in the fourth, we carry out a brief description on behalf of the theory of the fuzzy logic that will apply in this work; the fifth picks up the results that we have obtained when predicting the average price of Terra Networks and the number of securities traded in each Stock Exchange session for the company Abengoa; and lastly, in the sixth we expose the main conclusions that, in our opinion, are derived of this work.

2. TRADITIONAL SYSTEMS OF VALUATION

Knowing the value or price that will have a share in a determined period of time is the dream of any analyst or financial broker.

For that reason, there have had numerous tentative to obtain a model that would allow to predict the future rate of a share with the highest reliability. Among the models used to get this objective, we can mention the following ones:

- Until few years ago, the only well-known tool by the financial analysts of rent variable was that of the fundamental analysis. Fundamental analysis obtains its conclusions by a detailed study of the balance sheets, income statements, cash-flows statements, expansion plans, sales, future expectations of the company being studied, etc., as well as the sector based environment, the managerial competition, the news related to their operation, etc.

With all these components the fundamental analysis determines an intrinsic value of the company around which the share should quote on the Stock Exchange. An overvaluation or under valuation will indicate the investor or analyst if it is better for him to sell or buy some shares of that firm. To do it, the following models can be used:

- The approach or model of the earnings, which considers that the value of the shares will depend on the prospective profits and on the necessity of investing of the company in order to get such profits.
 - ✓ The Gordon model or approach of the dividend; in this case it is considered that the only explanatory variable are the dividends.
 - ✓ The approach of the investment opportunities; it is valid to value the shares that are in situation of losses.
 - ✓ The analysis of the balances by means of two fundamental techniques: percentage analysis and ratio analysis. In general, the first one is useful when you want to compare two companies of different size and/or to see the tendencies that these percentages have.

- As a perspective of support of the method previously mentioned, the analysts have also used the denominated *technical analysis* that is no more than the observation of the evolution of the markets starting from their graphic representation. This analysis, based on three premises (all that can affect the price of any securities has already been taken into account, the price movement and the history always repeats), tries to predict the tendency that will have the analysed value, in short as well as long term.
- The appearance in the Stock Exchange world of non statistic and econometrics specialised investors favoured the beginning of the use of the quantitative techniques in the financial world. This can be considered as an important qualitative change seeing that financial analysts had the possibility to use a tool that, although sometimes complex, is certainly powerful and satisfactory when the wanted results are obtained.

The quantitative analysis consists in using a group of mathematical, statistical or logical rules in order to build and later examine a mathematical representation of the real economic world that is known as a model. Therefore, the aim is to create an economic reality by means of an or several mathematical equations, a rule of conduct or a logical relationship in order to check, through quantitative techniques (i.e.: the analysis of econometrics) if the real data confirm the relation that had been established a priori.

3. THE PROBLEM OF THE TRADITIONAL SYSTEMS OF VALUATION FOR THE TECHNOLOGICAL VALUES

The companies that integrate what has been denominated New Economy are at present characterised by the fact that they are of recent creation, they do not make profit and proportionate volatile performance. This last characteristic is mainly

developed because it is known that these companies will revolutionise the world and therefore that, it would be very profitable to invest in some of them. However what is impossible to know is which will still exist in the future and dominate its sector. Logically, this fact causes some fluctuations of its price, as in the rising as in the fall. These ones can be really spectacular according to the variations of the investors' expectations.

Considering this situation, the estimate of these companies is quite difficult to give using the traditional methods (Gordon's model or the profits model). Since most of these does not have dividends, and generally, they even do not make or expect profit neither at short nor middle term. We should add that, although the activities that those firms have developed are considered as a novelty, we still do not know how to predict the rate of growth of the company. To know it, the analysts try to look for tools that would allow them to estimate these companies in function of the little information that is known about them, of the own environment in which they are moving and its future.

At present, as Mascareñas (2000) points out, the analysts have three types of financial instruments that allow to value the companies of Internet:

- *The discount of the future cash flows*; technique based on the intuitive idea that the value of a company for an investor or shareholders is the present value of the flows of money that will generate in the future.
- *The comparable companies*; method that is based on the search of another company that quotes in the stock market and that has the similar characteristic (same sector, similar size, future evolution, etc). The indicative ratios of the value are applied to this company in order to obtain the value of the company that we want to know. Next, and only for your information, we have collected some of the ratios and multipliers that can be used as an element of comparison to get the value of a company in this new sector:

- ✓ *Ratio of the sale's incomes*; it is determined dividing the value of the company in the market among the sale's incomes. The product of this ratio multiplied by the incomes of a company gives its theoretical value. An aspect that is necessary to define when we use this ratio is if the value of the company in the market refers to its market capitalization or the enterprise value (value of market of its assets).
- ✓ *The unique visitors' ratio*; in this case we take the value of the company in the market (market capitalization or enterprise valued) and we divide it by the number of unique visitors (number of people who visit a web in one month time), we obtain the ratio denominated multiplier of the unique visitors of the company. To get the market value of a determined company we would only have to multiply this ratio by the number of unique visitors of the company in question.
- ✓ *Multiplier of pages seen*; it is a similar ratio as the one described previously. It indicates us how the investors value the number of pages of a determined company seen. To determine it, we have to divide the value of the company in the market by the number of pages seen. Thus, if we multiply this ratio by the number of pages of the company seen we could obtain the current value of the assets.
- ✓ *Subscribers' multiplier and registered users*; it is mainly used to value the suppliers of internet services (ISP), the suppliers of services on line (OSP), the portals and the on line brokers.
- *Multiplier of the EBITDA*; this method is fundamentally used by the companies of telecommunications when they have reached the maturity (Time Warner, Disney or TCI). The supporting idea of this multiplication is that the value of the company is a ratio of its profit before paying-offs, interests and taxes (EBITDA).

- *Other ratios and relationships*; we can mention the relationship between the incomes and the costs of the e-marketing direct, the cost of clients' acquisition, the ratio of pop (places in which the physical connections to Internet are realised) and the relationship between the value of the company and its quota in the potential market, etc.
- *The methodology of the real options* (Brito, 2000); this approach is recommended for companies that present the following characteristics: their markets have a high level of uncertainty, they maintain some type of leadership in their sector and, they are managed by people who understand the options and have a certain ability to exercise them.

Besides the traditional techniques of valuation and those described previously, from the middle of the decade of the eighty, many investigators have begun to use the techniques of the neural networks to make predictions and to try to model the behavior of the quotations of the shares on the stocks markets; we have clear examples in the works of: White (1988), Yoon and Swales (1991), Kimoto et al (1990), Kamijo and Tanigawa (1990); or on the evolution of the Stock Exchange trend, Kin & King (2000) and Bosarge (1991).

The problem that, in our opinion, have these models is that the adaptation of the parameters does not reflect a result that allows to incorporate the knowledge of a human expert, in our case an investment advisor. This circumstance rebounds negatively in the efficiency of the model, since the result of the adjustment of the variables is difficultly interpretable.

Lately, like in the work of Hutchinson (1998), the fuzzy logic has been used to model dynamic systems, clearly not lineal, as the case of our work. The main advantage that the fuzzy logic incorporates -in our case the fuzzy neural network - is that it allows to incorporate the expert's knowledge in any moment, transforming the model into an open system. Furthermore, it allows that the adaptation of the

parameters responds to real and physically interpretable variables. These variables when belonging to a real range of values (universe of discourse) makes that the behavior of the model is adjusted, in all moment, to the reality that seeks to model.

Because of all that we said before, we have considered that it would be of great interest to find a model that allows to predict the market value of the companies that form the New Market. Among these, we have chosen Terra Networks, CORP., because it represents a clear example of the special characteristics that define these companies: high volatility in their share price and absence of profits and dividends.

4. FUZZY LOGIC KNOWLEDGE OF INPUT-OUTPUT DATA

A collection of fuzzy IF-THEN rules can be expressed by (Wang, 1994):

$$R^{(l)} : \text{IF } x_1 \text{ is } F_1^l \text{ and...and } x_n \text{ is } F_n^l \text{ THEN } y \text{ is } G^l \quad (1)$$

where $l=1,2,\dots,M$ is the number of rules. F_i^l are fuzzy sets of input variables in $U_i \subset R$. G^l are singleton membership functions in $V \subset R$. F_i^l y G^l are linguistic variables and U_i , V are universes of discourse. The input variables $x=(x_1,\dots,x_n)^T \in U_1 \times \dots \times U_n$ for $i=1,2,\dots,n$, where n is the number of input variables; $y \in V$ is the output variable.

The set of rules (1) with center average defuzzifier, product-inference rule and singleton fuzzifier are of the following form:

$$f(x) = \frac{\sum_{l=1}^M y^l \left[\prod_{i=1}^n \mu_{F_i^l}(x_i, \theta_i^l) \right]}{\sum_{l=1}^M \left[\prod_{i=1}^n \mu_{F_i^l}(x_i, \theta_i^l) \right]} \quad (2)$$

where $\mu_F(x, \theta)$ is the membership function $F \in U$ equivalent to a set of ordered pairs:

$$F = \{(x, \mu_F(x, \theta)) : x \in U\} \quad (3)$$

and $\theta_i^l = \alpha_i^l, \beta_i^l, \dots$ are the adjustable parameters of precedent part of rules and y^l is the consequent of rules. The equation (2) can be represented as a feedforward network shown in figure 1.

If we have an input-output pair $(x_1, x_2, \dots, x_n; y)$, the equation (2) can be minimized using the cost function:

$$J = \frac{1}{2}(f(x) - y)^2 = \frac{1}{2}(e)^2 \quad (4)$$

applying the back-propagation training algorithm. Then, the parameters of precedent part are updated using:

$$\theta_i^l(k+1) = \theta_i^l(k) - \eta \frac{\partial J(k)}{\partial \theta_i^l(k)} \quad (5)$$

In a similar way, the consequent is adjusted by:

$$y^l(k+1) = y^l(k) - \eta \frac{\partial J(k)}{\partial y^l(k)} \quad (6)$$

where η is the learning rate.

To obtain the update parameter law (5) it is necessary to apply the chain rule, that is:

$$\frac{\partial J(k)}{\partial \theta_i^l(k)} = e \frac{\frac{\partial A}{\partial \theta_i^l(k)} B - A \frac{\partial B}{\partial \theta_i^l(k)}}{B^2} \quad (7)$$

of course, the final update law depends on the membership function (Gaussian, Triangular, Bell or any derivative membership function).

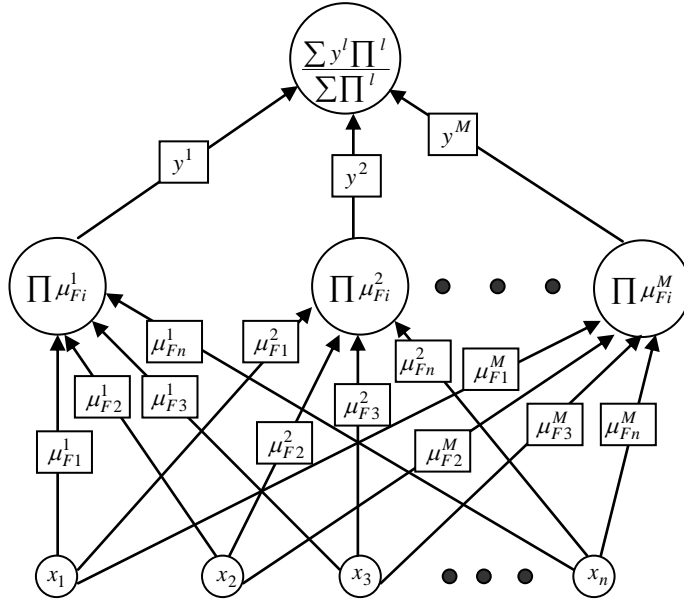


Figure 1.- Feedforward network representation of a fuzzy inference system.

5. AVERAGE PRICE AND STOCK TRANSACTIONS ESTIMATION (Andújar, 2000)

The share price evolution can be predicted using fuzzy rules, more specifically, the universal approximation equation (2).

The Stock Exchange of Madrid through the Service of Studies provided us, regarding the companies that integrate the New Market, among other the following economic, market and statistical variables: year 's results, dividend per share, closing price, average price, numbers of securities traded, volatility and beta coefficient.

Of among all the companies that integrate the one mentioned market, we have chosen Terra Networks since it is a clear example of company to which is not possible to apply the traditional systems of valuation. Starting from the data of the company and with the model based on the fuzzy logic, you try to predict the value of the average price for the following section which is based on the available historical information until the day before we try to predict it.

In the case of this company that was launched in the Stock Exchange on November 16th 1999, in order to realize our work we have used three variables, numbers of securities negotiated, average price and close price in 90 sessions (see appendix 1). The dividends and profits had not been taken into account because the former do not exist and the latter are negative. Other variables as the beta and the volatility have not been considered because they are constant values in the period in which the modeling has been developed and because they don't have a significant dynamic to obtain the model. In spite of what we said before, when historical data exist in a sufficient number, it would allow to incorporate it to the dynamic of the model and then these variables can be taken into account.

To model this company, we consider a black box (see figure 2) with the following input-output (closing price, stock transactions and average price) for each one of the sessions.

Figure 2 shows the input-output representation for average price prediction, where Δ represents the shift operator. The number of delayed values of variables are selected on modeling phase, considering the square average error and the nonlinear behavior.

A set of rules using the form (1) can be defined as:

$$R^{(l)} : \text{IF } C_{p1} \text{ is } F_1^l \text{ and } C_{p2} \text{ is } F_2^l \text{ and } S_{t1} \text{ is } F_3^l \text{ and } S_{t2} \text{ is } F_4^l \text{ and } A_{p1} \text{ is } F_5^l \text{ and } A_{p2} \text{ is } F_6^l \text{ THEN } A_{pt} \text{ is } G^l \quad (8)$$

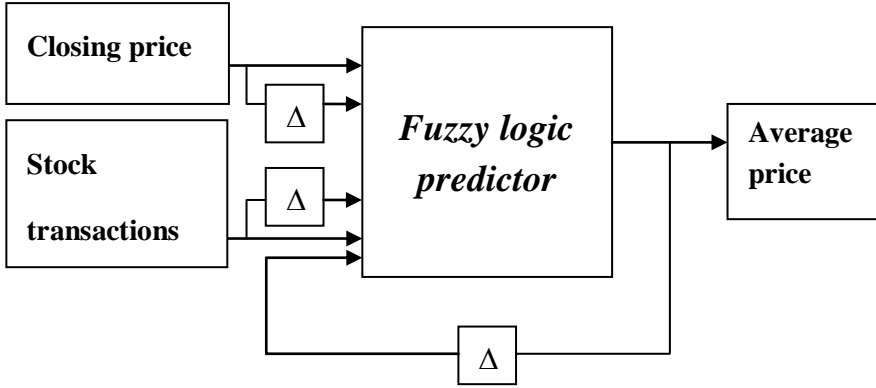


Figure 2.- Black box modeling of average price for Terra.

where C_{p1} is the previous closing price and C_{p2} is the actual closing price, S_{t1} is the delayed stocks transactions and S_{t2} is the actual stocks transactions, A_{p1} is the double delayed average price, A_{p2} is the delayed average price and A_{pt} is the average price that will be predicted.

Using the input-output data that shows the evolution of Terra, for a fuzzy system with 40 rules, an initial learning rate of 0.01 with momentum and 1000 epoch, a model can be obtained (see figure 3 and appendix 2).

The model shown in figure 3 is obtained from six variables selected in the precedent part of rules, where the membership function defined for the delayed stock transaction (S_{t1}) is shown in figure 4.

As we can see in figure 4, the membership functions show an irregular distribution of the center of Gauss function. That behavior is inadequate to extract the expert knowledge of fuzzy rules. A better result can be obtained using the Mars index in the phase of adaptation (that is, a gradient descent algorithm with restrictions).

The stocks transactions prediction can be useful for economic decision. In that case, the set of rules (8) can be defined by:

$$R^{(l)} : \text{IF } C_{p1} \text{ is } F_1^l \text{ and } C_{p2} \text{ is } F_2^l \text{ and } A_{p1} \text{ is } F_3^l \text{ and } A_{p2} \text{ is } F_4^l \text{ and } S_{t1} \text{ is } F_5^l \text{ and } S_{t2} \text{ is } F_6^l \text{ THEN } S_{pt} \text{ is } G^l \quad (9)$$

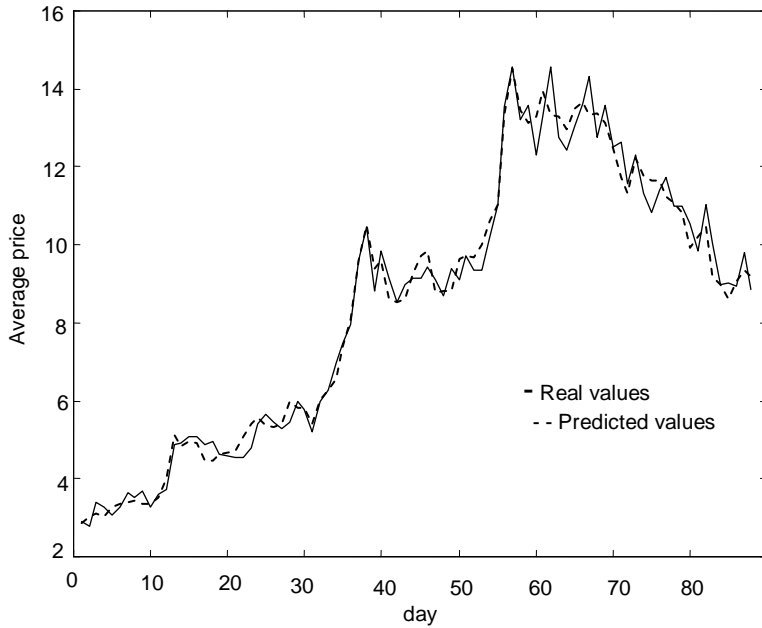


Figure 3.- Estimation of Terra average price.

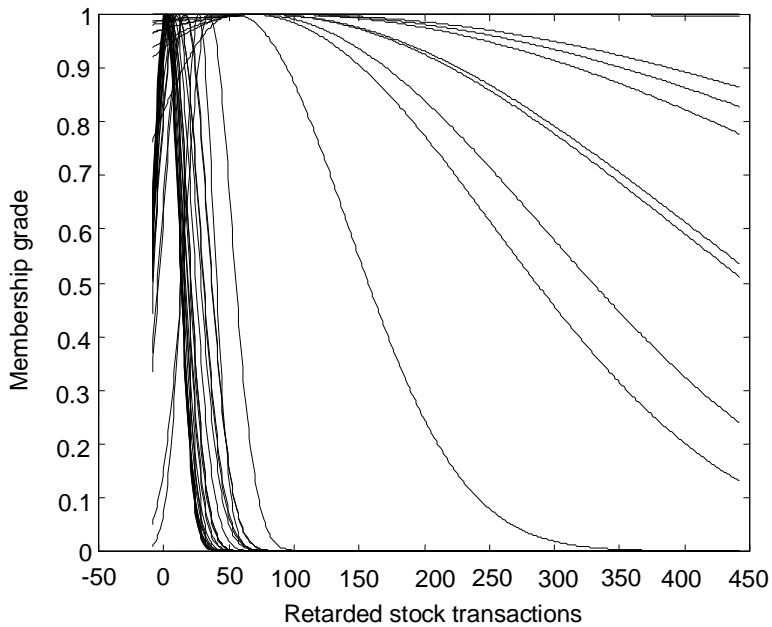


Figure 4.- Membership functions for delayed stock transactions (S_{t1}) variable.

A clear example of the open of our model (in the sense of their capacity of adaptation if the necessary knowledge is incorporated) can be seen in the figure 5. In this case, by means of the information picked up in the Service of Studies of the Stock Exchange of Madrid, the variable to estimate has been the number of securities traded in each stock exchange session for the company Abengoa. The model shown is a fuzzy system with 35 rules, a initial learning rate of 0.02 with momentum and 250 epoch.

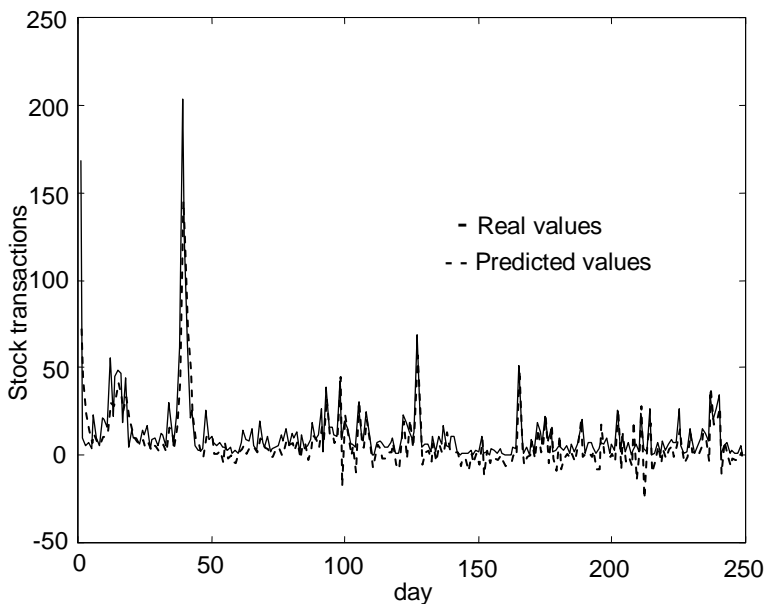


Figure 5.- Estimation of Abengoa stock transactions

6. CONCLUSIONS

The estimate of the average price of the Terra's stocks is more than acceptable, mainly if we take into account that we have only worked with three variables (average price, closing price and stocks transactions). This can be explained because of the specific characteristics of the stocks we have just mentioned. Although, the model does not end up predicting the average price with a 100%

reliability, it allows to estimate clearly which will be the tendency of the analysed stock, bullish or bearish.

Thanks to the characteristic of “universal approacher” that this model presents, it is possible to extend his application to other stocks (like is the case of Abengoa). For those that have a bigger number of variables, that would imply that the result of the variable to predict would be much more reliable.

In our opinion, our model of fuzzy neural network presents regarding the neural networks the following advantages:

- It allows to incorporate in any moment the knowledge of a human expert. Circumstance that transforms the model into an open system.
- The parameters that adapt respond to real and physically interpretable variables, which belong to a real range of values (universe of discourse), which makes that the behavior of the model is adjusted in every moment to the reality that seeks to model.

Lastly, we can highlight that the financial analysts can count on a quite reliable short-term prediction technique, if they have enough information.

APPENDIX 1

TERRA NETWORKS			
Date	Closing price	Average price	Stock transactions
19991116	0	0	40.305.357
19991117	37,00	29,50	28.386.481
19991118	31,45	32,70	9.535.121
19991119	28,02	28,67	3.932.908
19991122	30,80	30,14	3.045.750
19991123	30,31	30,97	2.298.761
19991124	30,00	30,07	2.400.068
19991125	32,40	32,59	5.200.513
19991126	32,90	33,62	2.417.755
19991129	34,10	34,02	3.403.503
19991130	34,21	34,40	1.316.138
19991201	33,58	33,73	3.686.156
19991202	33,42	33,54	1.860.120
19991203	36,40	35,33	2.676.515
19991207	41,86	40,17	2.663.908
19991209	48,82	51,08	3.129.569

19991210	49,73	48,40	1.888.940
19991213	49,40	49,50	827.577
19991214	48,85	48,98	999.440
19991215	43,75	44,58	1.835.758
19991216	45,49	44,70	1.740.745
19991217	45,04	46,13	1.226.725
19991220	46,80	46,85	1.119.146
19991221	46,50	46,97	685.315
19991222	51,45	50,77	2.147.827
19991223	53,50	54,28	1.476.330
19991227	54,50	55,80	982.890
19991228	52,25	53,73	591.908
19991229	53,15	53,32	621.092
19991230	54,25	54,24	687.192
20000103	58,35	59,70	1.757.645
20000104	56,75	58,00	9.816.912
20000105	57,65	58,12	1.688.486
20000107	56,10	54,20	1.944.545
20000110	59,60	60,25	1.467.406
20000111	65,00	62,67	2.021.105
20000112	67,95	65,62	1.777.395
20000113	73,80	73,79	3.064.948
20000114	80,95	80,89	2.370.940
20000117	97,90	96,08	2.246.356
20000118	103,50	105,04	3.822.079
20000119	92,50	94,01	2.562.566
20000120	90,60	95,81	1.637.964
20000121	83,75	85,92	1.865.377
20000124	85,00	85,08	2.430.217
20000125	86,00	86,13	1.357.330
20000126	93,15	92,57	1.986.060
20000127	99,00	97,36	2.169.733
20000128	94,50	98,24	6.979.294
20000131	87,00	87,80	3.090.694
20000201	85,70	88,14	1.236.693
20000202	89,40	88,13	1.477.385
20000203	98,90	96,46	1.924.263
20000204	96,05	97,01	1.322.236
20000207	95,15	96,87	1.163.572
20000208	102,50	100,03	2.129.992
20000209	106,85	106,14	1.572.946
20000210	119,25	110,47	2.124.734
20000211	137,10	133,20	2.922.982
20000214	138,00	145,39	2.706.803
20000215	126,60	134,68	1.897.925
20000216	131,45	131,26	1.618.676
20000217	136,60	133,00	897.750
20000218	139,25	139,28	1.495.863
20000221	137,30	133,39	961.540
20000222	127,00	132,85	932.915
20000223	130,40	129,54	726.019
20000224	130,90	134,95	783.052
20000225	139,75	136,67	661.658

20000228	132,60	133,19	917.128
20000229	135,00	133,80	677.145
20000301	128,65	131,30	797.193
20000302	123,95	124,82	1.166.155
20000303	117,50	117,20	1.547.305
20000306	120,00	113,39	1.705.266
20000307	121,25	122,36	1.454.978
20000308	117,35	117,86	759.886
20000309	113,00	116,44	1.023.537
20000310	117,15	116,62	738.900
20000313	109,75	112,29	1.057.245
20000314	112,20	110,61	703.692
20000315	107,10	108,40	674.455
20000316	94,80	99,10	2.135.589
20000317	107,70	102,09	1.481.846
20000320	99,85	104,41	1.270.681
20000321	90,95	91,95	2.201.338
20000322	85,00	89,97	1.908.341
20000323	85,50	86,05	1.592.364
20000324	93,55	90,47	1.396.545
20000327	90,70	93,40	714.754
20000328	92,00	91,66	465.197

APPENDIX 2

The rules variables exposed below (of the form (8)) have the following interpretation:

$$x1= C_{p1}; x2= C_{p2}; x3=S_{t1}; x4=S_{t2}; x5=A_{p1}; x6=A_{p2}$$

in the membership functions $F_j k$:

j = Number of rule; k = Number of input variable.

If $x1$ is F_{11} and $x2$ is F_{12} and $x3$ is F_{13} and $x4$ is F_{14} and $x5$ is F_{15} and $x6$ is F_{16} Then Apt is $2.196e+000x1$ $1.773e+000x2$ $1.896e+001x3$ $5.436e+000x4$ $1.211e+000x5$ $6.944e-001x6$

If $x1$ is F_{21} and $x2$ is F_{22} and $x3$ is F_{23} and $x4$ is F_{24} and $x5$ is F_{25} and $x6$ is F_{26} Then Apt is $-2.343e+000x1$ $-2.420e+000x2$ $9.435e-001x3$ $-7.012e-001x4$ $-2.556e+000x5$ $-3.339e+000x6$

If $x1$ is F_{31} and $x2$ is F_{32} and $x3$ is F_{33} and $x4$ is F_{34} and $x5$ is F_{35} and $x6$ is F_{36} Then Apt is $2.143e+000x1$ $2.021e+000x2$ $1.208e+001x3$ $3.749e+000x4$ $1.414e+000x5$ $8.696e-001x6$

If $x1$ is F_{41} and $x2$ is F_{42} and $x3$ is F_{43} and $x4$ is F_{44} and $x5$ is F_{45} and $x6$ is F_{46} Then Apt is $-6.484e-001x1$ $-6.210e-001x2$ $-5.758e+000x3$ $-2.684e+000x4$ $-9.942e-001x5$ $-1.769e+000x6$

If x_1 is F 5 1 and x_2 is F 5 2 and x_3 is F 5 3 and x_4 is F 5 4 and x_5 is F 5 5 and x_6 is F 5 6 Then Apt is $-9.418e-001x_1 - 8.710e-001x_2 - 7.716e+000x_3 - 3.146e+000x_4 - 1.155e+000x_5 - 2.001e+000x_6$

If x_1 is F 6 1 and x_2 is F 6 2 and x_3 is F 6 3 and x_4 is F 6 4 and x_5 is F 6 5 and x_6 is F 6 6 Then Apt is $4.493e+000x_1 4.010e+000x_2 2.882e+001x_3 1.071e+001x_4 3.272e+000x_5 2.978e+000x_6$

If x_1 is F 7 1 and x_2 is F 7 2 and x_3 is F 7 3 and x_4 is F 7 4 and x_5 is F 7 5 and x_6 is F 7 6 Then Apt is $2.184e+000x_1 1.958e+000x_2 1.199e+001x_3 3.169e+000x_4 1.448e+000x_5 8.725e-001x_6$

If x_1 is F 8 1 and x_2 is F 8 2 and x_3 is F 8 3 and x_4 is F 8 4 and x_5 is F 8 5 and x_6 is F 8 6 Then Apt is $1.482e+000x_1 1.401e+000x_2 4.997e+000x_3 1.772e+000x_4 9.217e-001x_5 2.513e-001x_6$

If x_1 is F 9 1 and x_2 is F 9 2 and x_3 is F 9 3 and x_4 is F 9 4 and x_5 is F 9 5 and x_6 is F 9 6 Then Apt is $-8.274e-001x_1 - 6.915e-001x_2 - 6.583e+000x_3 - 2.892e+000x_4 - 6.940e-001x_5 - 9.652e-001x_6$

If x_1 is F 10 1 and x_2 is F 10 2 and x_3 is F 10 3 and x_4 is F 10 4 and x_5 is F 10 5 and x_6 is F 10 6 Then Apt is $8.646e-001x_1 7.806e-001x_2 - 5.641e-002x_3 3.661e-001x_4 3.943e-001x_5 - 3.756e-001x_6$

If x_1 is F 11 1 and x_2 is F 11 2 and x_3 is F 11 3 and x_4 is F 11 4 and x_5 is F 11 5 and x_6 is F 11 6 Then Apt is $-8.203e-001x_1 - 7.565e-001x_2 - 6.673e+000x_3 - 3.343e+000x_4 - 1.088e+000x_5 - 1.920e+000x_6$

If x_1 is F 12 1 and x_2 is F 12 2 and x_3 is F 12 3 and x_4 is F 12 4 and x_5 is F 12 5 and x_6 is F 12 6 Then Apt is $-7.231e-001x_1 - 4.940e-001x_2 - 7.655e+000x_3 - 2.923e+000x_4 - 9.765e-001x_5 - 1.695e+000x_6$

If x_1 is F 13 1 and x_2 is F 13 2 and x_3 is F 13 3 and x_4 is F 13 4 and x_5 is F 13 5 and x_6 is F 13 6 Then Apt is $-3.140e-001x_1 5.558e-003x_2 - 5.645e+000x_3 - 2.408e+000x_4 - 6.877e-001x_5 - 1.261e+000x_6$

If x_1 is F 14 1 and x_2 is F 14 2 and x_3 is F 14 3 and x_4 is F 14 4 and x_5 is F 14 5 and x_6 is F 14 6 Then Apt is $-2.861e-001x_1 1.327e-001x_2 - 6.946e+000x_3 - 2.629e+000x_4 - 6.477e-001x_5 - 9.250e-001x_6$

If x_1 is F 15 1 and x_2 is F 15 2 and x_3 is F 15 3 and x_4 is F 15 4 and x_5 is F 15 5 and x_6 is F 15 6 Then Apt is $-1.295e+000x_1 - 9.689e-001x_2 - 6.947e+000x_3 - 3.167e+000x_4 - 1.216e+000x_5 - 1.725e+000x_6$

If x_1 is F 16 1 and x_2 is F 16 2 and x_3 is F 16 3 and x_4 is F 16 4 and x_5 is F 16 5 and x_6 is F 16 6 Then Apt is $-1.404e-001x_1 3.289e-001x_2 - 5.050e-001x_3 - 4.061e-003x_4 - 2.117e-001x_5 1.502e-001x_6$

If x_1 is F 17 1 and x_2 is F 17 2 and x_3 is F 17 3 and x_4 is F 17 4 and x_5 is F 17 5 and x_6 is F 17 6 Then Apt is $-1.384e-001x_1 5.000e-001x_2 - 1.050e+000x_3 5.449e-002x_4 - 1.829e-001x_5 3.082e-001x_6$

If x1 is F 18 1 and x2 is F 18 2 and x3 is F 18 3 and x4 is F 18 4 and x5 is F 18 5 and x6 is F 18 6
Then Apt is 1.525e-001x1 1.542e-001x2 -3.628e+000x3 -5.437e-001x4 2.603e-001x5 6.843e-002x6

If x1 is F 19 1 and x2 is F 19 2 and x3 is F 19 3 and x4 is F 19 4 and x5 is F 19 5 and x6 is F 19 6
Then Apt is -1.398e+000x1 -9.067e-001x2 -7.239e+000x3 -2.958e+000x4 -1.402e+000x5 -
1.589e+000x6

If x1 is F 20 1 and x2 is F 20 2 and x3 is F 20 3 and x4 is F 20 4 and x5 is F 20 5 and x6 is F 20 6
Then Apt is 1.602e-001x1 2.610e-001x2 -3.284e+000x3 -8.304e-001x4 2.449e-001x5 2.453e-001x6

If x1 is F 21 1 and x2 is F 21 2 and x3 is F 21 3 and x4 is F 21 4 and x5 is F 21 5 and x6 is F 21 6
Then Apt is -2.934e-001x1 1.656e-002x2 -1.597e+000x3 -2.291e-001x4 -1.767e-001x5 -1.124e-
001x6

If x1 is F 22 1 and x2 is F 22 2 and x3 is F 22 3 and x4 is F 22 4 and x5 is F 22 5 and x6 is F 22 6
Then Apt is -2.050e-001x1 3.324e-001x2 -9.358e-001x3 -8.279e-002x4 -2.212e-001x5 1.658e-
001x6

If x1 is F 23 1 and x2 is F 23 2 and x3 is F 23 3 and x4 is F 23 4 and x5 is F 23 5 and x6 is F 23 6
Then Apt is 2.418e-001x1 5.798e-001x2 -3.255e+000x3 -2.161e-001x4 3.790e-001x5 4.827e-001x6

If x1 is F 24 1 and x2 is F 24 2 and x3 is F 24 3 and x4 is F 24 4 and x5 is F 24 5 and x6 is F 24 6
Then Apt is -1.607e-001x1 4.060e-001x2 -2.953e-001x3 3.089e-001x4 -1.981e-001x5 2.644e-001x6

If x1 is F 25 1 and x2 is F 25 2 and x3 is F 25 3 and x4 is F 25 4 and x5 is F 25 5 and x6 is F 25 6
Then Apt is 2.687e-002x1 1.014e+000x2 -1.264e+000x3 -3.649e-001x4 -9.502e-002x5 8.639e-
001x6

If x1 is F 26 1 and x2 is F 26 2 and x3 is F 26 3 and x4 is F 26 4 and x5 is F 26 5 and x6 is F 26 6
Then Apt is 3.226e-001x1 1.161e+000x2 -1.595e+000x3 -5.561e-001x4 2.263e-001x5
1.032e+000x6

If x1 is F 27 1 and x2 is F 27 2 and x3 is F 27 3 and x4 is F 27 4 and x5 is F 27 5 and x6 is F 27 6
Then Apt is 1.387e-001x1 1.129e+000x2 -1.779e+000x3 -5.126e-001x4 6.263e-002x5 9.342e-001x6

If x1 is F 28 1 and x2 is F 28 2 and x3 is F 28 3 and x4 is F 28 4 and x5 is F 28 5 and x6 is F 28 6
Then Apt is 2.193e-001x1 1.232e+000x2 -1.854e+000x3 -5.383e-001x4 6.384e-002x5
1.024e+000x6

If x1 is F 29 1 and x2 is F 29 2 and x3 is F 29 3 and x4 is F 29 4 and x5 is F 29 5 and x6 is F 29 6
Then Apt is 5.671e-001x1 1.733e+000x2 -3.032e+000x3 -4.505e-001x4 2.984e-001x5
1.491e+000x6

If x1 is F 30 1 and x2 is F 30 2 and x3 is F 30 3 and x4 is F 30 4 and x5 is F 30 5 and x6 is F 30 6
Then Apt is 2.390e+000x1 2.267e+000x2 4.122e-001x3 3.534e+000x4 1.980e+000x5 1.163e+000x6

If x_1 is F 31 1 and x_2 is F 31 2 and x_3 is F 31 3 and x_4 is F 31 4 and x_5 is F 31 5 and x_6 is F 31 6
Then Apt is $-3.098e-001x_1 -1.119e-001x_2 -5.253e+000x_3 -3.265e+000x_4 -2.482e-002x_5 -3.163e-001x_6$

If x_1 is F 32 1 and x_2 is F 32 2 and x_3 is F 32 3 and x_4 is F 32 4 and x_5 is F 32 5 and x_6 is F 32 6
Then Apt is $1.310e-001x_1 9.509e-001x_2 -9.700e-001x_3 1.616e-001x_4 -1.167e-005x_5 6.379e-001x_6$

If x_1 is F 33 1 and x_2 is F 33 2 and x_3 is F 33 3 and x_4 is F 33 4 and x_5 is F 33 5 and x_6 is F 33 6
Then Apt is $1.019e+000x_1 2.038e+000x_2 -3.073e+000x_3 -7.156e-001x_4 6.203e-001x_5 1.714e+000x_6$

If x_1 is F 34 1 and x_2 is F 34 2 and x_3 is F 34 3 and x_4 is F 34 4 and x_5 is F 34 5 and x_6 is F 34 6
Then Apt is $2.326e+000x_1 3.235e+000x_2 -9.856e-001x_3 2.769e-001x_4 2.077e+000x_5 2.131e+000x_6$

If x_1 is F 35 1 and x_2 is F 35 2 and x_3 is F 35 3 and x_4 is F 35 4 and x_5 is F 35 5 and x_6 is F 35 6
Then Apt is $6.009e-001x_1 7.510e-001x_2 -2.500e+000x_3 -1.551e+000x_4 9.180e-002x_5 -4.176e-001x_6$

If x_1 is F 36 1 and x_2 is F 36 2 and x_3 is F 36 3 and x_4 is F 36 4 and x_5 is F 36 5 and x_6 is F 36 6
Then Apt is $-1.048e+000x_1 -5.501e-001x_2 -2.817e+000x_3 -1.080e+000x_4 -1.559e+000x_5 -1.559e+000x_6$

If x_1 is F 37 1 and x_2 is F 37 2 and x_3 is F 37 3 and x_4 is F 37 4 and x_5 is F 37 5 and x_6 is F 37 6
Then Apt is $6.000e-001x_1 1.260e+000x_2 -4.757e+000x_3 -1.152e+000x_4 1.053e-001x_5 4.709e-001x_6$

If x_1 is F 38 1 and x_2 is F 38 2 and x_3 is F 38 3 and x_4 is F 38 4 and x_5 is F 38 5 and x_6 is F 38 6
Then Apt is $2.408e+000x_1 2.910e+000x_2 3.148e-001x_3 5.267e-001x_4 2.401e+000x_5 2.867e+000x_6$

If x_1 is F 39 1 and x_2 is F 39 2 and x_3 is F 39 3 and x_4 is F 39 4 and x_5 is F 39 5 and x_6 is F 39 6
Then Apt is $2.972e-001x_1 5.149e-001x_2 -8.358e+000x_3 -2.530e+000x_4 1.951e-001x_5 -2.754e-001x_6$

If x_1 is F 40 1 and x_2 is F 40 2 and x_3 is F 40 3 and x_4 is F 40 4 and x_5 is F 40 5 and x_6 is F 40 6
Then Apt is $4.475e+000x_1 4.325e+000x_2 2.435e+000x_3 1.839e+000x_4 4.481e+000x_5 4.245e+000x_6$

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